Focus on Physics

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The Delightful Catenary Curve

When teaching how *tension* and *compression* relate to geometrical structures such as bridges, arches, and domes, I show a picture of the Notre Dame Cathedral in Paris (Figure 1A), completed in the 14th century.

I point out the elaborate buttresses that keep the walls from pushing outward while supporting its weight. Architects of the day had not yet learned how to hold up a very large, massive building without external propping. This was accomplished in the 17th century in the construction of St. Paul's Cathedral in London (Figure 1B).

Why, I ask, is St. Paul's Cathedral free of such buttresses? Aha, inside its famous dome is an inner "secret dome" that provides structural support. To understand this, let's first investigate the roles of tension and compression in structures.

Tension

I stretch a length of rope taut, explaining that the stretching force we call *tension* acts in a direction parallel to the direction of the rope. When I let the rope sag between my hands, tension vectors within the sagging rope continue to align with the rope. The curved shape of the sag is determined by this alignment of tension vectors. Likewise for a sagging chain or sagging cable.

A rope, chain, or cable supported at its ends and hanging only by its own weight takes the shape of a special curve called a *catenary.* I sketch a sagging chain on the board and show that tension vectors between links of the chain are everywhere parallel to the curve with no components of tension perpendicular to the curve (Figure 2). The chain ends can be held at different distances apart, making the curve deep or shallow. As long as the chain supports only its own weight, it's a catenary.

FIGURE 1

A. Notre Dame Cathedral, Paris. B. St. Paul's Cathedral, London.

FIGURE 2

Tension between links in the chain align with (are parallel to) the curve of the chain. The curve is a catenary.

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If a sagging chain or cable supports weight that is distributed uniformly in a horizontal direction, as is approximately true in a suspension bridge, then the shape of the curve is a *parabola*, the same curve followed by a tossed ball. The curved cables of a suspension bridge or suspended roadway are approximately parabolas. Only if the cable supports only its own weight—such as sagging clotheslines, power lines, and strands of spider webs—is the shape a catenary.

Compression—and the inverted catenary (an arch)

Of particular interest is an inverted catenary, where internal forces are of *compression* rather than tension. When a free-standing arch takes the shape of an inverted catenary, the weight of the arch is supported by compression forces pressing along the arch's curve. There are then no compressive forces perpendicular to the curve. My grandson Manuel delightfully shows two catenaries in

FIGURE 3

Curves of the sagging chain held by Manuel and of the Gateway Arch in St. Louis are catenaries.

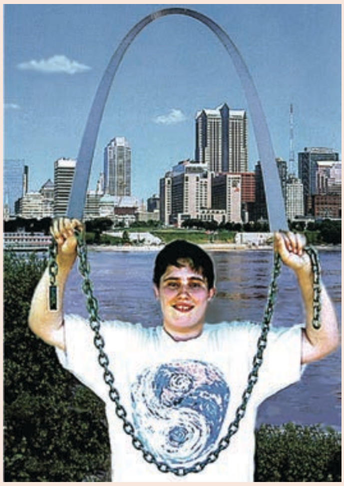

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Figure 3, one of a suspended chain and in the background the Gateway Arch in St. Louis, Missouri.

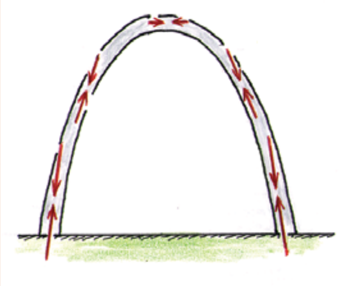
I sketch the Gateway Arch, showing that compression vectors between adjacent slabs that make up the arch are everywhere parallel to the curve (Figure 4). I tell students that they could make a stable mini-arch out of slippery blocks of ice if the shape of the arch is a catenary! But if the shape were any other, such as a semicircle, blocks of ice would squeeze free, and the arch would collapse. Where strength is important, modern arches are usually catenaries.

The three-dimensional catenary: a dome

I ask students to imagine rotating an arch through a complete circle. I then help them to reason that the result would be a dome. Just as for an arch, the weight of any dome produces compression, which tends to collapse the dome unless the compression forces are aligned with (parallel to) the dome's curve—in other words, a catenary.

FIGURE 4

Compression vectors between slabs are everywhere parallel to the Gateway Arch.



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Christopher Wren, the brilliant architect who designed St. Paul's Cathedral, believed that a *hemispheric dome* (half of a sphere) would be the most beautiful but knew it would be too weak by itself to hold the extremely heavy lantern structure he planned to place atop it. So, he cleverly designed three nesting domes: a hemispheric outer dome, a steeper inner dome, and—hidden in the middle—a dome that approximated a catenary and provided the necessary structural support (Figure 5).

Modern domes that span vast areas without the interruption of supporting columns are most often three-dimensional catenaries. There are shallow domes (the Jefferson Memorial) and tall ones (the United States Capitol). The catenary principle is employed in the Houston Astrodome, the roof of Washington Dulles International Airport, and the igloos in the Arctic.

The catenary egg

To relate this physics concept to the natural world, I introduce students to the shape of a common egg. People have always wondered why crushing an egg by squeezing along its long axis is so difficult (Figure 6), while a chickie can easily poke its way out from the inside. Penetrating the shell from inside deals with tension rather than compression. Only the weaker shell tension must be overcome. But what makes the compressive forces on the outside so strong? Can you guess? I ask. (And of course they can.)

FIGURE 5

The nesting domes of St. Paul's Cathedral.

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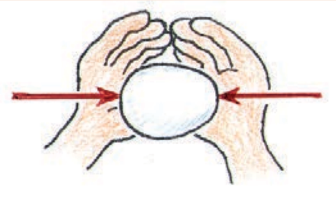
A double catenary

I direct student attention to the two most curved surfaces of an egg: its two ends. With an egg held so that first one end, then the other, is on top, they see that a chain follows the contour of the

FIGURE 6

Why does squeezing an egg along its long axis fail to break the egg?



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egg—at each end (Figure 7). Aha! Students are delighted to see that both ends of the egg, one more strongly curved than the other, are catenaries. Nature has not overlooked the catenary!

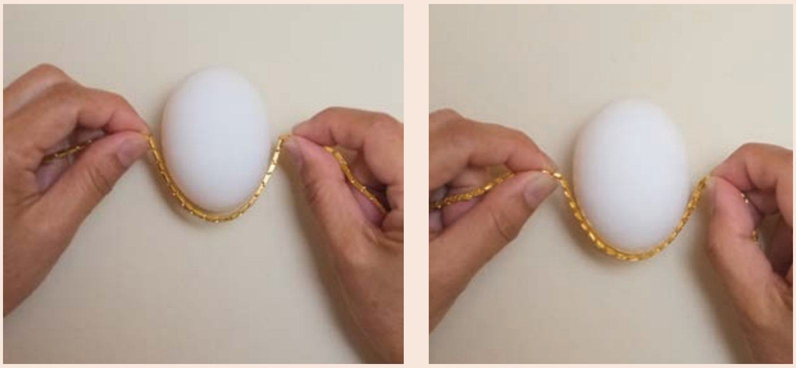
Paul G. Hewitt (pghewitt@aol.com) *is the author of the popular textbook* Conceptual Physics 12th edition *and coauthor with his daughter Leslie Hewitt and nephew John Suchocki of* Conceptual Physical Science 6th edition, *Pearson Education.*

On the web

For more on the catenary: *http://bit.ly/catenaries*. Tutorial screencast lessons by the author are on *www.HewittDrewIt.com* and *www.ConceptualAcademy.com*.

FIGURE 7

A chain follows the curves of an egg—catenaries at both ends.



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